Answer on Question#50337 - <Math> - <Other>

Find the length of the parametric curve $\begin{cases} x(t) = \cos^3 t \\ y(t) = \sin^3 t \end{cases}, \quad 0 < t < \frac{\pi}{2}$

Solution. Since, the length of the parametric curve is $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$ then:

$$L = \int_0^{\pi/2} \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = \int_0^{\pi/2} \sqrt{(-3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}\,dt = \int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\,dt =$$

$$= 3\int_0^{\pi/2} \sqrt{\cos^2 t \sin^2 t \left(\cos^2 t + \sin^2 t\right)}\,dt = 3\int_0^{\pi/2} \sqrt{\cos^2 t \sin^2 t}\,dt = 3\int_0^{\pi/2} \left|\cos t \sin t\right| dt = 3\int_0^{\pi/2} \cos t \sin t\,dt =$$

$$= 3\int_0^{\pi/2} \sin t\,d(\sin t) = 3\frac{\sin^2 t}{2}\Bigg|_0^{\pi/2} = \frac{3}{2}$$

Answer: The length of the parametric curve $\begin{cases} x(t) = \cos^3 t \\ y(t) = \sin^3 t \end{cases}, \quad 0 < t < \frac{\pi}{2}$ is $\frac{3}{2}$