Answer on Question #50334 – Math – Other

Question.

Show that
$\ln(1+x) < x$

Solution.

Proof by contradiction.

Let use the Mean Value Theorem: if a function f is continuous on the closed interval $[a, b]$, where $a < b$, and differentiable on the open interval (a, b), then there exists a point c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

In our case,

$$f(x) = \ln(1+x) - x$$

For $x = 0$: $f(x) = 0$

For $x = 1$: $f(x) = \ln(2) - 1 < 0$

For $x \in (0; \infty)$:

$$f'(c) = \frac{f(x) - f(0)}{x - 0} = \frac{f(x)}{x} = \frac{\ln(1+x)}{x} - 1 < 0$$

But, it can be negative. It's contradiction. So,

$$\ln(1+x) < x$$