

Answer on Question #50307 – Math - Algebra

$xp+3yp=2(z-x^2p^2)$ solve this equation.

Solution

$$xp + yp = 2(z - x^2p^2)$$

Solve for x

(other values of y, z, p should be known in order to get a definite solution):

$2p^2x^2 + px + (yp - 2z) = 0$ is a quadratic equation with respect to x , when $p \neq 0$;

its discriminant is $D1 = p^2 - 4 \cdot 2p^2(yp - 2z) = p^2 - 8yp^3 + 16zp^2$

Solutions are the following:

If $p \neq 0$, then

$$x = \frac{-p \pm \sqrt{D1}}{2 \cdot 2p^2} = \frac{-p \pm \sqrt{1 - 8yp + 16z}}{4p^2} = -\frac{1}{4p} - \frac{1}{4p} \sqrt{1 - 8yp + 16z}; -\frac{1}{4p} + \frac{1}{4p} \sqrt{1 - 8yp + 16z} .$$

If $p = 0$, then $0 = 2z$, hence

$x = C$ is an arbitrary real number, $y = E$ is an arbitrary real number, $z = 0$.

Solve for y

(other values of x, z, p should be known in order to get a definite solution):

$xp + yp = 2(z - x^2p^2)$ is a linear equation with respect to y .

If $p \neq 0$, then $xp + yp = 2(z - x^2p^2) \Rightarrow$ |subtract xp from both sides| \Rightarrow

$$\Rightarrow yp = 2(z - x^2p^2) - xp \Rightarrow$$
 |divide both sides by p | \Rightarrow

$$y = \frac{2(z - x^2p^2) - xp}{p} = \frac{2z}{p} - 2x^2p - x$$

If $p = 0$, then $0 = 2z$, hence

$y = E$ is an arbitrary real number, $x = C$ is an arbitrary real number, $z = 0$.

Solve for z

(other values of x, y, p should be known in order to get a definite solution):

$xp + yp = 2(z - x^2p^2)$ is a linear equation with respect to z ;

$$\Rightarrow$$
 |add $2x^2p^2$ to both sides| $\Rightarrow xp + yp + 2x^2p^2 = 2z \Rightarrow$ |divide both sides by 2| \Rightarrow
$$\frac{xp + yp + 2x^2p^2}{2} = z, \text{ i.e. } z = \frac{xp + yp + 2x^2p^2}{2}$$

Solve for p

(other values of x, y, z should be known in order to get a definite solution):

$2x^2p^2 + (x + y)p - 2z = 0$ is a quadratic equation with respect to p , when $x \neq 0$;

its discriminant is $D2 = (x + y)^2 - 4 \cdot 2x^2 \cdot (-2z) = x^2 + y^2 + 2xy + 16x^2z$

Solutions are the following:

If $x \neq 0$, then

$$p = \frac{-(x+y) \pm \sqrt{D2}}{2 \cdot 2x^2} = \frac{-(x+y) \pm \sqrt{x^2 + y^2 + 2xy + 16x^2z}}{4x^2} = \frac{-(x+y) - \sqrt{x^2 + y^2 + 2xy + 16x^2z}}{4x^2}; \frac{-(x+y) + \sqrt{x^2 + y^2 + 2xy + 16x^2z}}{4x^2}.$$

If $x = 0$, then $yp = 2z$ is a linear equation with respect to p , hence

If $x = 0$ and $y \neq 0$, then $p = \frac{2z}{y}$;

If $x = 0$ and $y = 0$ and $z \neq 0$, then the equation does not have solutions

If $x = 0$ and $y = 0$ and $z = 0$, then the equation has infinitely many solutions, i.e. $p = K$, where K is an arbitrary real constant.

Note that there are infinitely many solutions when two or less values of x, y, z, p are known, they should satisfy condition $z \geq \frac{xp + yp}{2}$.