Answer on Question #50248 – Math – Complex Analysis

1) Given:

$$\sum_{n=0}^{\infty} \frac{2^n}{i^n}(z+\pi)^n$$

Find:

the Radius of convergence of the power series

Solution:

$$a_n = \frac{2^n}{i^n}$$

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{2^n}{i^n}\cdot\frac{i^{n+1}}{2^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{2^n \cdot i \cdot i^n}{i^n \cdot 2 \cdot 2^n}\right| = \left|\frac{i}{2}\right| = \frac{1}{2}$$

Answer: $R = \frac{1}{2}$

2) Given:

$$f(z) = z^2 + \frac{2}{z^3} - 2$$

Determine:

whether $z_0 = \infty$ is a singularity of $f(z)$

if its singularity, classify it

Solution:

a) function $f(z)$ is not analytic at the point $z_0 = \infty$, so $z_0 = \infty$ is a singularity of $f(z)$

b) Function $g(z) = f\left(\frac{1}{z}\right) = \frac{1}{z^2} + 2z^3 - 2$ has a singularity $z = 0$, which is a pole of order 2 , hence function $f(z)$ has a singularity $z_0 = \infty$, which is a pole of order 2.

$$\lim_{z\to\infty}(z^2 + \frac{2}{z^3} - 2) = \lim_{z\to\infty}\frac{z^5 - 2z^3 + 2}{z^3} = \infty \qquad \Rightarrow \qquad z_0 = \infty \text{ is a pole}$$

Answer: $z_0 = \infty$ is a singularity of $f(z)$, $z_0 = \infty$ is a pole.