nswer on Question #50234 Math, Complex Analysis

1) **Given**:

$$\sum_{n=0}^{\infty}\frac{2^n}{i^n}(z+\pi)^n$$

Find:

the Radius of convergent of the power series

Solution:

$$a_n=\frac{2^n}{i^n}$$

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{2^n}{i^n}\cdot\frac{i^{n+1}}{2^{n+1}}\right|=\lim_{n\to\infty}\left|\frac{2^n\cdot i\cdot i^n}{i^n\cdot 2\cdot 2^n}\right|=\left|\frac{i}{2}\right|=\frac{1}{2}$$

Answer: $R=\frac{1}{2}$

2) **Given**:

$$f(z)=z^2+\frac{2}{z^3}-2$$

Determine:

whether $z_0=\infty$ is a singularity of $f(z)$

if its singularity, classify it

Solution:

a) function $f(z)$ is not analytic in the point $z_0=\infty$, so

$z_0=\infty$ is a singularity of $f(z)$

b) $\lim_{z\to\infty}(z^2+\frac{2}{z^3}-2)=\lim_{z\to\infty}\frac{z^5-2z^3+2}{z^3}=\infty \quad\Rightarrow\quad z_0=\infty$ is a pole of order 2,

because $f(z)=z^2+\frac{2}{z^3}-2=z^2\left(1+\frac{2}{z^5}-\frac{2}{z^2}\right)=z^m g(z)$, where m – is the order of pole, when $g(\infty)\neq 0$.

Answer: $z_0=\infty$ is a singularity of $f(z)$, $z_0=\infty$ is a pole of order 2.