

Answer on Question #50232, Math, Complex Analysis

Find the sum of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot (z - \pi/2)^2$ afterward compute the sum of the series $\sum_{n=0}^{\infty} \frac{1}{(2n)!}$

Solution:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (z - \pi/2)^2 = (z - \pi/2)^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$$

It is known that Taylor series for $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, where $|x| < \infty$

$$\text{So } \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} = -\frac{1}{1!} + \frac{1}{2!} - \frac{1}{4!} + \dots = \frac{e^{j1} + e^{-j1}}{2j} = \cos(1), \text{ then}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (z - \pi/2)^2 = (z - \pi/2)^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} = (z - \pi/2)^2 \cos(1)$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n)!} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{4!} + \dots = \frac{e^1 + e^{-1}}{2} = \cosh(1)$$

$$\text{Answer: } \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (z - \pi/2)^2 = (z - \pi/2)^2 \cos(1) \quad ; \quad \sum_{n=0}^{\infty} \frac{1}{(2n)!} = \cosh(1)$$