Answer on Question #50227 – Math – Complex Analysis

Decide whether the series is convergent or divergent

A)) ∑ [8^{ n+i.(2^-n) }] / [9 ^ n]

B)) ∑ Conjugate all of this ([n+in+(n^i)] / [i ^ n])

Solution

A) $\sum \frac{8^{n+i2^{-n}}}{9^n} = \left(\frac{8}{9}\right)^n 8^{i2^{-n}} \sum \frac{8^{n+i2^{-n}}}{9^n} = \sum \left(\frac{8}{9}\right)^n 8^{i/2^n}.$

By Cauchy criterion

$$q = \lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{8}{9}\right)^n 8^{i/2^n}\right|} = \frac{8}{9} < 1,$$ hence the series is convergent.

B) $\sum \overline{\left(\frac{n+in+n^i}{i^n}\right)} = \sum \overline{\left(\frac{n+in+e^{i\ln n}}{i^n}\right)}$. By Cauchy criterion

$$q = \lim_{n\to\infty} \sqrt[n]{\left|\frac{n+in+e^{ilnn}}{i^n}\right|} = \lim_{n\to\infty} |n+in+e^{ilnn}|$$

$$= \lim_{n\to\infty} \sqrt{(n+\cos(lnn))^2 + (n+\sin(lnn))^2} = +\infty,$$ hence

the series is divergent.