Answer on Question #50216, Math, Algebra

Find the algebraic relation in x, y, z if $\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$ are in A.P. Show that if x, y, z are in A.P. then x=y=z.

Solution:

We start with the definition. Arithmetic progression is a sequence where $T_{n+1} - T_n = d, n \geq 1$. Common difference d of the Arithmetic progression and T_n is the n^{th} term.

Thus we will consider the following relation in x, y, z if $\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$.

$$\tan^{-1}x + \tan^{-1}z = 2\tan^{-1}y$$

From the noted expression we will apply the formula for determination Tangents of sums.

$$\tan^{-1}\left[\frac{x+z}{1-xz}\right] = \tan^{-1}[\frac{2y}{1-y^2}]$$

Now we can write the following.

$$\frac{x+z}{1-xz} = \frac{2y}{1-y^2}$$

$$\frac{2y}{1-xz} = \frac{2y}{1-y^2}$$

We have to note that $x+z=2y$ since x, y and z are in Arithmetic progression.

$$1-xz = 1-y^2$$

Now we can simplify the obtained expression.

$$-xz = 1-1-y^2$$

$$xz = y^2$$

We know that the xz equal to

$$xz = (\frac{x+z}{2})^2$$

From the noted above equation $x+z=2y$ we can express the y.

$$y = \frac{x+z}{2}$$

Simplify the expression of xy by substitution y.

$$xz = (\frac{x+z}{2})^2 = \frac{(x+z)^2}{2^2}$$

Now we can multiply the term of xz by 4.

$$4xz = (x + z)^2$$

$$4xz - x^2 - 2xz - z^2 = 0$$

$$-x^2 + 2xz - z^2 = 0$$

Multiply all terms by -1 and simplify.

$$x^2 - 2xz + z^2 = 0$$

Another words we can change this expression into known formula.

$$(x - z)^2 = 0$$

This mean x is equal to z or we can write $x = z$. Since $x + z = 2y \;\; => 2x = 2y$ and $x = y$.

Finally we can write that $x = y = z$.