Given:

$$\sum_{n=2}^{\infty} \sqrt[2]{\sqrt[n]{\sqrt[n]{\sqrt[n]{\cos n + i \sin n}}}}$$

Decide:

if these series is convergent or divergent

Solution:

$$a_n = (\cos n + i \sin n)^{\frac{1}{2n^3}} = \left(e^{in}\right)^{\frac{1}{2n^3}} = e^{i \cdot \frac{\arg(e^{in}) + 2\pi k}{2n^3}} \qquad k \in Z$$

$$\arg\left(e^{in}\right) = n$$

$$|a_n| = 1$$

we can use the fact

$$\lim_{n \to \infty} a_n = 0 \quad \Leftrightarrow \quad \lim_{n \to \infty} |a_n| = 0$$

we obtain

$\lim_{n \to \infty} |a_n| = 1 \neq 0$ the necessary condition of convergence of series is not executed, so

the series is divergent.

Answer: divergent