Question #50128, Math, Complex Analysis

The formula $\sum_{n\geq 0} w^n = \frac{1}{1-w}$, for $|z|<1$ will be used for $w=-z^2$:

$$\sum_{n\geq 2}(-1)^n z^{2n+3} = z^7(-1)^2\sum_{n\geq 0}(-1)^n z^{2n} = z^7\frac{1}{1+z^2}$$

The power series $\sum_{n\geq 2}(-1)^n i^{2ni} i^{3i}$ has the terms

$$(-1)^n i^{2ni} i^{3i} = (-1)^n (e^{i\pi/2})^{2ni}(e^{i\pi/2})^{3i} = (-e^{-\pi})^n e^{-1.5\pi}$$

So, for $w=-e^{-\pi}$ and first formula

$$\sum_{n\geq 2}(-1)^n i^{2ni} i^{3i} = e^{-1.5\pi}e^{-2\pi}\sum_{n\geq 0}(-e^{-\pi})^n = e^{-3.5\pi}\sum_{n\geq 0} w^n = \frac{e^{-3.5\pi}}{1-w} = \frac{e^{-3.5\pi}}{1+e^{-\pi}} = \frac{e^{-2.5\pi}}{e^{\pi}+1}$$