Answer on Question #50126, Math, Complex Analysis

Given:

$$B_n = \frac{Ln(2-n)}{(|n+i\sqrt{2}|)^2} \qquad n \geq 3$$

Decide:
whether these series is convergent or divergent

Solution:

$$B_n = \frac{Ln(2-n)}{(|n+i\sqrt{2}|)^2} = \frac{Ln|2-n| + i(\arg(2-n)+2\pi k)}{(\sqrt{n^2+2})^2}$$

$$n \geq 3 \quad \Rightarrow \quad 2-n \leq 0 \quad \Rightarrow \quad \arg(2-n) = \pi$$

Ln is a principle value, so $k = 0$

$$B_n = \frac{Ln(n-2)+i\pi}{n^2+2} \qquad |B_n| = \frac{\sqrt{Ln^2(n-2)+\pi^2}}{n^2+2} \sim \frac{Ln(n-2)}{n^2}$$

Using the integral test for convergence by Maclaurin Cauchy

$$\int_3^{+\infty} \frac{Ln(x-2)}{x^2}dx = \left\{u = Ln(x-2), v' = \frac{1}{x^2}, u' = \frac{1}{x-2}, v = -\frac{1}{x}\right\} =$$

$$= -\frac{Ln(x-2)}{x}\Bigg|_3^{+\infty} + \int_3^{+\infty} \frac{1}{x(x-2)}dx = (\frac{Ln(x-2)}{x} + \frac{1}{2}[Ln(x-2) - Ln(x)])\Bigg|_3^{+\infty} =$$

$$= \left(-\frac{Ln(x-2)}{x} + \frac{1}{2}Ln\frac{x-2}{x}\right)\Bigg|_3^{+\infty} = \frac{Ln(3-2)}{3} - \frac{1}{2}\frac{Ln(3-2)}{3} = -\frac{1}{2}Ln\frac{1}{3} \quad \in R$$

Answer: convergent