Answer on Question #50125 - Math - Complex Analysis

Given:

$$\sum_{n=2}^{\infty} \sqrt[n]{\sqrt[n]{\sqrt[n]{\cos n + i \sin n}}}$$

Decide:

if this series is convergent or divergent

Solution:

$$a_n = (\cos n + i \sin n)^{\frac{1}{n^3}} = \left(e^{in}\right)^{\frac{1}{n^3}} = e^{i\frac{\arg(e^{in}) + 2\pi k}{n^3}} \qquad k \in \mathbb{Z}$$

$$\arg(e^{in}) = n$$

$$|a_n| = 1$$

we can use the fact

$$\lim_{n\to\infty} a_n = 0 \quad \Leftrightarrow \quad \lim_{n\to\infty} |a_n| = 0$$

we obtain

 $\lim_{n\to\infty} |a_n| = 1 \neq 0$ the necessary condition of convergence of series is not executed, so the series is divergent.

Answer: divergent