

Answer on Question #49980 – Math – Statistics and Probability

You are having three coins. First coin has two tails, second coin has two heads and the third one has one head and one tail. You choose a coin at random and toss, and get tail. What is the probability that coin chosen is two tailed coin?

(A) $1/2$

(B) $1/3$

(C) $2/3$

(D) $1/4$

Solution

Method 1.

First coin has two tails (2 chances), second coin has two heads (0 chances) and the third one has one head and one tail (1 chance). Totally we have 3 chances to get a tail from 6 possibilities.

If we get a tail, the probability, that coin chosen is two tailed, will be

$$P = \frac{\text{Number of chances to get a tail with two tailed coin}}{\text{Total number of chances to get a tail}} = \frac{2}{3}.$$

Method 2.

Hypotheses:

$H_1 =$ "the first coin with two tails was chosen",

$H_2 =$ "the second coin with two heads was chosen",

$H_3 =$ "the third coin with one head and one tail was chosen".

Event $A =$ "To get a tail when a random coin is tossed".

If we choose a coin at random, then H_1, H_2, H_3 have equal probabilities, taking into account that $P(H_1) + P(H_2) + P(H_3) = 1$, we obtain that

$$P(H_1) = P(H_2) = P(H_3) = \frac{1}{3}.$$

According to statement of question, conditional probabilities are the following:

$$P(A|H_1) = 1, P(A|H_2) = 0, P(A|H_3) = \frac{1}{2}.$$

Evaluate the next probability by total probability formula:

$$\begin{aligned} P(A) &= \sum_{i=1}^n P(H_i)P(A|H_i) = P(H_1)P(A|H_1) + P(H_2)P(A|H_2) + P(H_3)P(A|H_3) = \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot \frac{1}{2} = \\ &= \frac{1}{3} \left(1 + \frac{1}{2} \right) = \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2} \end{aligned}$$

If it is known that the event A has occurred but it is unknown which of the events H_1, H_2, H_3 has occurred, then Bayes` formula is used:

$$P(H_k|A) = \frac{P(H_k)P(A|H_k)}{P(A)} = \frac{P(H_k)P(A|H_k)}{\sum_{i=1}^n P(H_i)P(A|H_i)}$$

In our problem, search for probability

$$P(H_1|A) = \frac{P(H_1)P(A|H_1)}{P(A)} = \frac{\frac{1}{3} \cdot 1}{\frac{1}{2}} = \frac{2}{3}$$

Answer: (C) $\frac{2}{3}$.