Answer on Question #49925 – Math – Calculus

If f(x,y)=1/3x^3+4xy-9x- y''^2

a) find the local extreme and saddle point of f(x,y)

b) evaluate ∫ from 0 to 1 * ∫ from x^2 to 2x (x^3+4y) dy dx

Solution

a) $f(x,y) = \frac{1}{3x^3} + 4xy - 9x - y^2$

$$\frac{\partial f(x,y)}{\partial x} = -\frac{3}{3x^4} + 4y - 9 - 0 = -\frac{1}{x^4} + 4y - 9$$

$$\frac{\partial f(x,y)}{\partial y} = 0 + 4x - 0 - 2y = 4x - 2y$$

$$\frac{\partial^2 f(x,y)}{\partial x^2} = \frac{4}{x^5} + 0 + 0 = \frac{4}{x^5}$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} = 0 - 2 = -2$$

$$\frac{\partial^2 f(x,y)}{\partial y \partial x} = \frac{\partial^2 f(x,y)}{\partial x \partial y} = 4 - 0 = 4$$

Let us find locale extreme and saddle points.

$$\begin{cases} \frac{\partial f(x,y)}{\partial x} = 0 \\ \frac{\partial f(x,y)}{\partial y} = 0 \end{cases}$$

$$\begin{cases} -\frac{1}{x^4} + 4y - 9 = 0 \\ 4x - 2y = 0 \end{cases}$$

From 2nd equation.

$$y = 2x$$

Put in 1st equation.

$$-\frac{1}{x^4} + 8x - 9 = 0$$

Only one real solution.

$$x_1 \approx 1.188$$

$$y_1 \approx 2.376$$

Second partial derivative test.

$$\begin{vmatrix} \frac{4}{x^5}\Big|x_1 & 4 \\ 4 & -2 \end{vmatrix} = -\frac{8}{x_1^5} - 16 < 0$$

, hence, point (x_1, y_1) – saddle point.

b) $\int_0^1 \int_{x^2}^{2x} (x^3 + 4y)\, dy\, dx$

$$\int_{x^2}^{2x} (x^3 + 4y)\, dy = \left(x^3 y + \frac{4y^2}{2}\right)\Big|_{x^2}^{2x} = (x^3 y + 2y^2)\Big|_{x^2}^{2x} =$$

$$(x^3(2x) + 2(2x)^2) - (x^3 x^2 + 2(x^2)^2) = (2x^4 + 8x^2) - (x^5 + 2x^4) = 8x^2 - x^5$$

$$\int_0^1 \int_{x^2}^{2x} (x^3 + 4y)\, dy\, dx = \int_0^1 \left(\int_{x^2}^{2x} (x^3 + 4y) dy\right) dx =$$

$$= \int_0^1 (8x^2 - x^5) dx = \left(\frac{8x^3}{3} - \frac{x^6}{6}\right)\Big|_0^1 = \left(\frac{8(1)^3}{3} - \frac{(1)^6}{6}\right) - \left(\frac{8(0)^3}{3} - \frac{(0)^6}{6}\right) = \frac{8}{3} - \frac{1}{6} = \frac{15}{6}.$$

Answer: a) $(1.188; 2.376)$ – saddle point. b) $\frac{15}{6}$.