Answer on Question #49923 – Math – Multivariable Calculus

Find the point of maximum and minimum for the function z=f(x,y)=x^2-xy+y^2+3x-2y+1.

Solution

Partial derivatives of the function are:

$$\frac{\partial f}{\partial x} = 2x - y + 3, \quad \frac{\partial f}{\partial y} = -x + 2y - 2,$$

$$\frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial^2 f}{\partial y^2} = 2, \quad \frac{\partial^2 f}{\partial x \partial y} = 1.$$

Let find critical points of the function.

$$\begin{cases} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \end{cases}, \quad \begin{cases} 2x - y + 3 = 0 \\ -x + 2y - 2 = 0 \end{cases}, \quad \begin{cases} y = 2x + 3 \\ -x + 2(2x+3) - 2 = 0 \end{cases}, \quad \begin{cases} y = 1/3 \\ x = -4/3 \end{cases}.$$

The critical point is $A\left(-\frac{4}{3}; \frac{1}{3}\right)$.

$$\Delta = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = 2 \cdot 2 - 1^2 = 3.$$

$$\Delta\Big|_A = 3 > 0, \quad \frac{\partial^2 f}{\partial x^2}\bigg|_A = 2 > 0, \text{ so } f_A = f_{\min} = \frac{16}{9} + \frac{4}{9} + \frac{1}{9} - 4 - \frac{2}{3} + 1 = -\frac{4}{3}.$$

Answer: $z_{\min} = f\left(-\frac{4}{3}; \frac{1}{3}\right) = -\frac{4}{3}$.