Answer on Question #49889-Math-Statistics and Probability

1) Let X be a random variable with density function as follows:

$$f(x) = \begin{cases} 2(\mathrm{x}-1), 1 < \mathrm{x} < 2 \\ 0, \text{elsewhere} \end{cases}$$

Find the expectation of x, and the variance.

If $g(x) = x^2 + x - 2$, where x has the same above density function.

1. $E(g(x))$.

2. $E(2)$.

Solution

The expectation of X is

$$E(X) = \int_1^2 x \cdot 2(\mathrm{x}-1)\, dx = 2\int_1^2 (x^2 - x)\, dx = 2\left(\frac{x^3}{3} - \frac{x^2}{2}\right)_1^2 = \frac{5}{3}.$$

The variance of X is

$$Var(X) = E(X^2) - E(X)^2.$$

$$E(X^2) = \int_1^2 x^2 \cdot 2(\mathrm{x}-1)\, dx = 2\int_1^2 (x^3 - x^2)\, dx = 2\left(\frac{x^4}{4} - \frac{x^3}{3}\right)_1^2 = \frac{17}{6}.$$

$$Var(X) = \frac{17}{6} - \left(\frac{5}{3}\right)^2 = \frac{1}{18}.$$

If $g(x) = x^2 + x - 2$, where x has the same above density function.

1.

$$E(g(x)) = \int_1^2 (x^2 + x - 2) \cdot 2(\mathrm{x}-1)\, dx = 2\int_1^2 (x^3 - 3x + 2)\, dx = 2\left(\frac{x^4}{4} - 3\frac{x^2}{2} + 2x\right)_1^2 = \frac{5}{2}.$$

2.

$$E(2) = \int_1^2 2 \cdot 2(\mathrm{x}-1)\, dx = 4\left(\frac{x^2}{2} - x\right)_1^2 = 2.$$