Question:

Solve the equation for exact solutions over the interval $[0, 2\pi)$. $4\sin^2 x + 8\sin x + 4 = 0$.

Solution:

Let $\sin x = t$, then we obtain the equation:

$4t^2 + 8t + 4 = 0,$

This equation has one root:

$D = b^2 - 4ac = 8^2 - 4 \cdot 4 \cdot 4 = 0,$

$t = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-8}{2 \cdot 8} = -1.$

So, let's back to the substitution $\sin x = t$:

$\sin x = -1.$

$x = \dfrac{3\pi}{2} + 2\pi n, n \in Z$

So, we can see that in the interval $[0, 2\pi)$ the equation $4\sin^2 x + 8\sin x + 4 = 0$ has one solution:

$x = \dfrac{3\pi}{2}.$

Answer:

$x = \dfrac{3\pi}{2}.$