# Answer on Question #49851 – Mathematics – Calculus

**Question:**

The gamma function of( -1) and of (0)?

**Solution:**

The Gamma function can be defined as an improper integral

$$\Gamma(\alpha) = \int_0^{+\infty} t^{\alpha-1} e^{-t} dt. \qquad (1)$$

This formula is valid for Re $\alpha>0$. For Re $\alpha<0$ and $1$-Re$\alpha>0$ the function $\Gamma(z)$ can be calculated by the formula

$$\Gamma(\alpha) = \frac{\pi}{\sin(\pi\cdot\alpha)\cdot\Gamma(1-\alpha)}. \qquad (2)$$

As we see, for integer Re$\alpha\leq 0$ the multiplier $\sin(\pi\cdot\alpha) = 0$ and the Gamma function is discontinuous (fig1.). So, $\alpha = -n$, $n=0, 1, 2$... are the points of discontinuity for $\Gamma(\alpha)$.

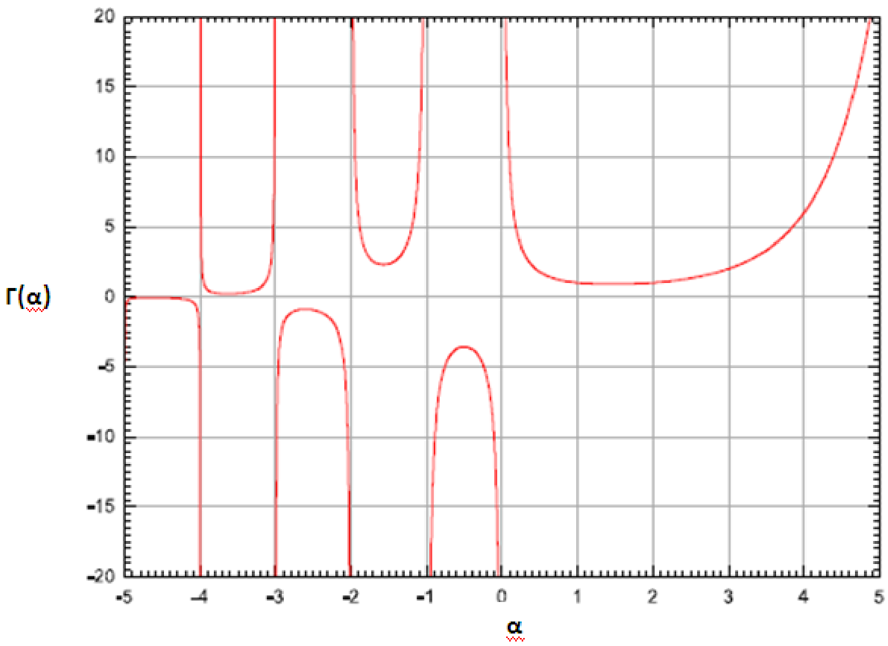


Fig 1. *The gamma function of the real argument*

Therefore, the Gamma function is continuous everywhere (for $Re\alpha>0$ and for $Re\alpha<0$) except $\alpha=-n$, where n ∈ Z. Using the reduction formula for Gamma function

$$\Gamma(\alpha) = \frac{\Gamma(\alpha+1)}{\alpha}. \qquad (3)$$

we can write

$$\Gamma(+0)=\lim_{\alpha\to+0}\Gamma(\alpha)=\lim_{\alpha\to+0}\frac{\Gamma(\alpha+1)}{\alpha}=\frac{\Gamma(1)}{\lim\limits_{\alpha\to+0}\alpha}=\frac{1}{\lim\limits_{\alpha\to+0}\alpha}=+\infty;$$

$$\Gamma(-0)=\lim_{\alpha\to-0}\Gamma(\alpha)=\lim_{\alpha\to-0}\frac{\Gamma(\alpha+1)}{\alpha}=\frac{\Gamma(1)}{\lim\limits_{\alpha\to-0}\alpha}=\frac{1}{\lim\limits_{\alpha\to-0}\alpha}=-\infty;$$

$$\Gamma(-1+0)=\lim_{\alpha\to-1+0}\Gamma(\alpha)=\lim_{\alpha\to-1+0}\frac{\Gamma(\alpha+1)}{\alpha}=\frac{\Gamma(+0)}{-1}=-\infty;$$

$$\Gamma(-1-0)=\lim_{\alpha\to-1-0}\Gamma(\alpha)=\lim_{\alpha\to-1-0}\frac{\Gamma(\alpha+1)}{\alpha}=\frac{\Gamma(-0)}{-1}=+\infty$$

and so on.

You can also look at our video on this topic:

https://www.youtube.com/watch?v=yu9k2iPta-k

**Answer:** $\alpha=-1$ and $\alpha=0$ are the points of discontinuity for $\Gamma(\alpha)$.