Answer on Question #49819 – Math – Statistics and Probability

Five university students were given an English achievement test before and after receiving instruction in basic grammar. Their scores are shown below:

Student	Before	After
A	20	18
B	18	22
C	17	15
D	16	17
E	12	9

Should we conclude that future students would show higher scores after instruction? Use the .05 significance level. Use hypothesis testing.

Solution

$n = n_1 = n_2 = 5.$

The mean difference is

$$\bar{d} = \overline{(x_a - x_b)} = \frac{\sum(x_a - x_b)}{n} = \frac{(18-20)+(22-18)+(15-17)+(17-16)+(9-12)}{5} = -0.4.$$

Sample standard deviation of difference is

$$\sum(x_a - x_b)^2 = (18-20)^2 + (22-18)^2 + (15-17)^2 + (17-16)^2 + (9-12)^2 = 34$$

$$s_d = \sqrt{\frac{\sum(x_a - x_b)^2 - n\bar{d}^2}{n-1}} = \sqrt{\frac{34 - 5\cdot(-0.4)^2}{5-1}} = 2.881.$$

Hypotheses:

$H_o: \bar{d} \leq 0; H_a: \bar{d} > 0.$

Decision Rule:

$\alpha = 0.05$

Degrees of freedom $n - 1 = 5 - 1 = 4$

Critical t-score from t-table $t^* = 2.132.$

Reject H_0 if $t > 2.132.$

Test Statistic:

$$t = \frac{\bar{d}}{\frac{s}{\sqrt{n}}} = \frac{-0.4}{\frac{2.881}{\sqrt{5}}} = -0.310.$$

Decision (in terms of the hypotheses):

Since $t = -0.310 < \text{t}^* = 2.132$ we fail to reject H_0.

Conclusion (in terms of the problem):

There is no sufficient evidence at 0.05 significance level that future students would show higher scores after instruction.