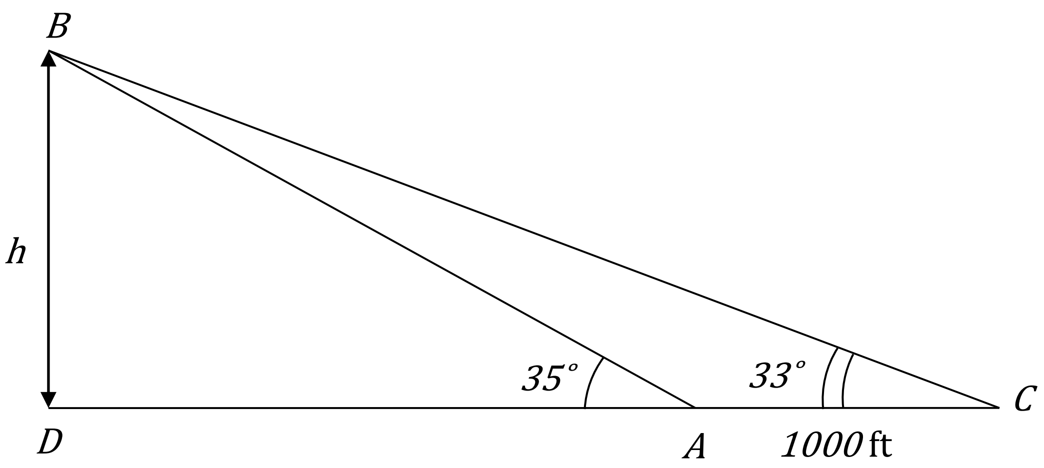
Answer on Question #49749 - Math - Trigonometry

As you walk on a straight level path toward a mountain, the measure of the angle of elevation to the peak of the mountain from one point is 33 degrees. From a point 1000 ft closer, the angle of elevation is 35 degrees. How high is the mountain?

Solution:

$\angle BAD = 35°$, $\angle BCA = 33°$, $AC = 1000$



Let h be the height of the mountain. Let's consider the triangle ABC. The sum of angles $\angle ABC = \hat{B}$ and $\angle ACB = \hat{C}$ equals the value of angle $\angle BAD$, because $\hat{A} + \hat{B} + \hat{C} = 180°$, where $\hat{A} = \angle BAC$, $\hat{A} + \angle BAD = 180°$, hence $\angle BAD = 180° - \hat{A} = \hat{B} + \hat{C}$.

From $\angle BAD = \hat{B} + \hat{C}$ we easily obtain that $\hat{B} = \angle BAD - \hat{C} = 35° - 33° = 2°$. Using the law of sines for triangle ABC we obtain

$$\frac{AC}{\sin \hat{B}} = \frac{BA}{\sin \hat{C}}$$

Solving this equation for BA we obtain

$$BA = AC\frac{\sin \hat{C}}{\sin \hat{B}} \qquad (1)$$

Now consider the triangle ABD. Using the law of sines for this triangle we obtain

$$\frac{BD}{\sin \angle BAD} = \frac{BA}{\sin \widehat{D}}$$

Solving this equation for BD and considering the fact that $\sin \widehat{D} = 1$ ($\widehat{D} = 90°$) we obtain

$$BD = BA \sin \angle BAD$$

Substitution of BA from the equation (1) gives us the following

$$h = BD = BA \sin \angle BAD = AC \frac{\sin \hat{C}}{\sin \hat{B}} \sin \angle BAD = 1000 \text{ ft} \cdot \frac{\sin 33^\circ}{\sin 2^\circ} \sin 35^\circ = 8951 \text{ ft}$$

Answer: 8951 ft.