Answer on Question #49747 – Math – Differential Calculus | Equations

Solve for i(t) for the circuit, given that V(t) = 10 sin5t V, R=4 W and L =2 H. (Ri + L di/dt = v)

Solution.

$$Ri + L\frac{di}{dt} = v \rightarrow \ 2\frac{di}{dt} + 4i = 10sin5t \ \rightarrow \ \frac{di}{dt} + 2i = 5sin5t$$

Multiply both sides by e^{2t} and obtain

$$e^{2t}\frac{di}{dt} + 2e^{2t}i = 5e^{2t}sin5t \rightarrow \ \frac{d}{dt}(e^{2t}i) = 5e^{2t}sin5t \ \rightarrow$$

Integrate both sides with respect to t

$$\rightarrow \ e^{2t}i = \int 5e^{2t}sin5tdt$$

To take integral $\int e^{2t}sin5tdt$, use 2 times integration by parts:

$$f = sin5t, \ g = \frac{1}{2}e^{2t}, \ df = 5cos5tdt, \ dg = e^{2t}dt$$

So, $\int e^{2t}sin5tdt = \frac{1}{2}e^{2t}sin5t - \frac{5}{2}\int e^{2t}cos5tdt$

Then $f = cos5t, \ g = \frac{1}{2}e^{2t}, \ df = -5sin5tdt, \ dg = e^{2t}dt$

So $\int e^{2t}sin5tdt = \frac{1}{2}e^{2t}sin5t - \frac{5}{2}\left(\frac{1}{2}e^{2t}cos5t + \frac{5}{2}\int e^{2t}sin5tdt\right) =$

$$= \frac{1}{2}e^{2t}sin5t - \frac{5}{4}e^{2t}cos5t - \frac{25}{4}\int e^{2t}sin5tdt \ \rightarrow$$

$$\left(1 + \frac{25}{4}\right)\int e^{2t}sin5tdt = \frac{1}{2}e^{2t}sin5t - \frac{5}{4}e^{2t}cos5t \rightarrow$$

$$\rightarrow \ \int e^{2t}sin5tdt = \frac{4}{29}\left(\frac{1}{2}e^{2t}sin5t - \frac{5}{4}e^{2t}cos5t\right)$$

Thus, $e^{2t}i = \frac{5}{29}(-5cos5t + 2sin5t)e^{2t} + c \rightarrow$

$\rightarrow$ $i(t) = ce^{-2t} - \frac{25}{29}cos5t + \frac{10}{29}sin5t$, where c is an arbitrary real constant.