Answer on Question #49554 – Math – Trigonometry

If A and B are acute angles satisfying $\sin A = \sin B \sin B$ and $2\cos A \cos A\ = 3\cos B \cos B$ then A+B=

Solution

$$\sin B \sin B + \cos B \cos B = 1 = \sin A + \frac{2}{3}\cos A \cos A = \sin A + \frac{2}{3}(1 - \sin^2 A).$$

$$\frac{2}{3}\sin^2 A - \sin A + \frac{1}{3} = 0 \rightarrow \sin^2 A - \frac{3}{2}\sin A + \frac{1}{2} = 0.$$

$$D = \left(-\frac{3}{2}\right)^2 - 4 \cdot \frac{1}{2} = \frac{1}{4}.$$

$$\sin A = \frac{\frac{3}{2} \pm \frac{1}{2}}{2} = 1\ ;\ \frac{1}{2}.$$

But A is acute angles, so $\sin A \neq 1$ $(A \neq 90°)$.

Thus,

$$\sin A = \frac{1}{2}\ and\ A = 30°.$$

$$\sin A = \sin B \sin B = \frac{1}{2} \rightarrow \sin B = \frac{1}{\sqrt{2}}\ (\ angle\ B\ is\ acute\ too) \rightarrow B = 45°.$$

So,

$$\mathrm{A} + \mathrm{B} = 75°.$$

Answer: $\mathbf{75}°$.