If $2 \tan A + \cot A = \tan B$ then $\cot A + 2 \tan(A - B) = ?$

Solution.

$$\cot A + 2 \tan(A - B) = \frac{1}{\tan A} + 2 \frac{\sin(A - B)}{\cos(A - B)} = \frac{1}{\tan A} + 2 \frac{\sin A \cos B - \sin B \cos A}{\cos A \cos B + \sin A \sin B} =$$

$$= \frac{1}{\tan A} + 2 \frac{(\sin A \cos B - \sin B \cos A):(\sin A \sin B)}{(\cos A \cos B + \sin A \sin B):(\sin A \sin B)} = \frac{1}{\tan A} + 2 \frac{\cot B - \cot A}{\cot A \cot B + 1} =$$

$$= \frac{\cot A \cot B + 1 + 2 \tan A (\cot B - \cot A)}{\tan A (\cot A \cot B + 1)} = \frac{\cot B (2 \tan A + \cot A) - 2 \tan A \cot A + 1}{\tan A (\cot A \cot B + 1)} =$$

$$= \frac{\cot B \tan B - 1}{\tan A (\cot A \cot B + 1)} = \frac{1 - 1}{\tan A (\cot A \cot B + 1)} = 0.$$