Answer on Question #49512 – Math – Differential Calculus | Equations

Solve the equation G (d^2)y/dx^2 - W(1-x)=0 where G and W are constants, subject to the conditions that

y(0)=0 , y'(0)=1 (1)

Solution

We have equation:

$$G \cdot \frac{d^2y}{dx^2} - W(1-x) = 0, y(0) = 0, y'(0) = 1$$

$$G \cdot \frac{d^2y}{dx^2} = W(1-x)$$

Divide both sides by G:

$$\frac{d^2y}{dx^2} = \frac{W}{G}(1-x) \quad (2)$$

Integrate (2) with respect to x:

$$\frac{dy}{dx} = \frac{W}{G}\left(x - \frac{x^2}{2}\right) + c_1 \quad (3)$$

Use the second initial condition (1) and previous equality (3):

$$y'(0) = 1 = \frac{W}{G}(0-0) + c_1 = c_1$$

So

$$c_1 = 1. \quad (4)$$

Take into account (4) and integrate both sides of (3) with respect to x:

$$y(x) = \frac{W}{G}\left(\frac{x^2}{2} - \frac{x^3}{6}\right) + x + c_2 \quad (5)$$

Use the first initial condition (1) and previous equality (5):

$$y(0) = 0 = \frac{W}{G}(0-0) + 0 + c_2 = c_2$$

So

$$c_2 = 0.$$

Answer: $y(x) = \frac{W}{G}\left(\frac{x^2}{2} - \frac{x^3}{6}\right) + x.$