Answer on Question #49505 – Math – Trigonometry

Question:

If $A+B+C=180^{\circ}$, prove that, $a\cdot sin(B-C)+b\cdot sin(C-A)+c\cdot sin(A-B)=0$

$$a\cdot sin(B-C)+b\cdot sin(C-A)+c\cdot sin(A-B)=0. \quad (1)$$

Solution:

Let us prove given trigonometric identity in too steps.

1) Use the sine of difference identity for the angles α and β

$$\sin(\alpha-\beta)=sin\alpha\cdot cos\beta-cos\alpha\cdot sin\beta. \quad (2)$$

Rewriting (1) by taking into account (2) we get

$$a\cdot\left(sinB\underline{cosC}-\underline{\underline{cosB}}sinC\right)+b\cdot\left(sinC\underline{\underline{\underline{cosA}}}-\underline{cosC}sinA\right)+c\cdot\left(sinA\underline{\underline{cosB}}-\underline{\underline{\underline{cosA}}}sinB\right)=$$
$$cosC(asinB-bsinA)+cosB(csinA-asinC)+cosA(bsinC-csinA). \quad (*)$$

2) Since $A+B+C=180^{\circ}$, then we can use the law of sines (an equation relating the lengths of the sides of any shaped triangle to the sines of its angles):

$$\frac{a}{sinA}=\frac{b}{sinB}=\frac{c}{sinC}. \quad (3)$$

Hence, we obtain the following relations

$$asinB=bsina,\ csinA=asinC,\quad bsinC=csinB. \quad (4)$$

Substituting (4) into (*) we see that

$$asinB-bsinA=csinA-asinC=bsinC-csinA=0.$$

Therefore, we get

$$a\cdot sin(B-C)+b\cdot sin(C-A)+c\cdot sin(A-B)=0.$$

QED.

Answer: $a\cdot sin(B-C)+b\cdot sin(C-A)+c\cdot sin(A-B)=0.$