Answer on Question #49501 – Math – Statistics and Probability

Ten voters were picked at random from those who voted in favour of a certain proposition and twelve from those who voted against it. The following figures give their ages:

In favour 28 33 27 31 29 25 50 30 25 41

Against 31 43 49 32 40 41 48 30 29 39 42 36

At 5% percent level of significance, it is evidence from the data that the mean age of those voting against the proposition is significantly different from the mean of those voting for it?

Solution

$H_0: \mu_1 = \mu_2;\ H_a: \mu_1 \neq \mu_2.$

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{28 + 33 + 27 + 31 + 29 + 25 + 50 + 30 + 25 + 41}{10} = 31.9.$$

$$s_1 = \sqrt{\frac{\sum x_1^2 - n_1 \bar{x}_1^2}{n_1 - 1}}.$$

$$\sum x_1^2 = 28^2 + 33^2 + 27^2 + 31^2 + 29^2 + 25^2 + 50^2 + 30^2 + 25^2 + 41^2 = 10735.$$

$$s_1 = \sqrt{\frac{10735 - 10 \cdot 31.9^2}{9}} = 7.88.$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{31 + 43 + 49 + 32 + 40 + 41 + 48 + 30 + 29 + 39 + 42 + 36}{12} = 38.3.$$

$$s_2 = \sqrt{\frac{\sum x_2^2 - n_2 \bar{x}_2^2}{n_2 - 1}}.$$

$$\sum x_2^2 = 31^2 + 43^2 + 49^2 + 32^2 + 40^2 + 41^2 + 48^2 + 30^2 + 29^2 + 39^2 + 42^2 + 36^2 = 18142.$$

$$s_2 = \sqrt{\frac{18142 - 12 \cdot 38.3^2}{11}} = 7.00.$$

We assume that the variances are equal.

Test statistic is

$$T = \frac{\bar{x}_2 - \bar{x}_1}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where S_p^2 is the pooled sample variance:

$$S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}.$$

$$S_p = \sqrt{\frac{(10-1)\cdot 7.88^2 + (12-1)\cdot 7^2}{10+12-2}} = 7.41.$$

So,

$$T = \frac{38.3 - 31.9}{7.41\sqrt{\frac{1}{10} + \frac{1}{12}}} = 2.017.$$

Critical value of test statistic 5% level of significance and $n_1 + n_2 - 2 = 10 + 12 - 2 = 20$ degrees of freedom from t-table is t^* =2.086.

We don't reject H_0 because test statistic is lower than critical value ($T = 2.017 < t^* = 2.086$).

There is no sufficient evidence from the data at 5% level of significance that the mean age of those voting against the proposition is significantly different from the mean of those voting for it.