Answer on Question #49483 – Math – Complex Analysis

Find if these sequence convergent or divergent (details)

1) Zn = [i.(z^n) - n.(3^(n+1))] / [i.n .(2^(n-1))]

2) [conjugate (4 n^2- i n +1)] / [(i n +3) ^2]

3) [8 ^ (n+1) - 5^(n)] / [5 . (8^n)+ 3 ^(n+1)]

Solution

We say that a complex series converges if and only if both the real and imaginary parts converge.

1) $z_n = \frac{iz^n - n3^{n+1}}{in2^{n-1}} = \frac{2}{n}\left(\frac{z}{2}\right)^n - \frac{6}{i}\left(\frac{3}{2}\right)^n = \frac{2}{n}\left(\frac{z}{2}\right)^n + 6i\left(\frac{3}{2}\right)^n = \frac{2}{n}\left(\frac{rcos\varphi + irsin\varphi}{2}\right)^n + 6i\left(\frac{3}{2}\right)^n =$
$= \frac{2}{n}\left(\frac{r^n\cos(n\varphi)}{2^n} + \frac{r^n\sin(n\varphi)}{2^n}\right) + 6i\left(\frac{3}{2}\right)^n.$

Since $\frac{3}{2} > 1$, $\left(\frac{3}{2}\right)^n \nrightarrow 0$, as $n \longrightarrow \infty$, because $\left(\frac{3}{2}\right)^n \to \infty$ as $n \to \infty$ (the necessary condition of convergence does not hold true), therefore sequence $6\left(\frac{3}{2}\right)^n$ is divergent, hence the imaginary part diverges and consequently sequence z_n diverges. Note that if $|z| \le 2$, then $\frac{2}{n}\left(\frac{z}{2}\right)^n \to 0$, as $n \to \infty$ (because $\frac{2}{n} \to 0$ as $n \to \infty$, and $\left(\frac{z}{2}\right)^n$ is bounded sequence in this case). If $|z| > 2$, then $\frac{2}{n}\left|\frac{z}{2}\right|^n$ diverges as $n \to \infty$ by d\`Alembert\`s ratio test (its real subsequence diverges, therefore $\frac{2}{n}\left(\frac{z}{2}\right)^n$ diverges).

2) $\frac{\overline{4n^2 - \iota n + 1}}{(\iota n+3)^2} = \frac{4n^2 + in + 1}{-n^2 + 6in + 9} = \frac{4 + \frac{i}{n} + \frac{1}{n^2}}{-1 + 6\frac{i}{n} + 9\frac{1}{n^2}} \to \frac{4}{-1} = -4$ as $n \to \infty$, because $\frac{i}{n} \to 0$ as $n \to \infty$, $\frac{1}{n^2} \to 0$ as $n \to \infty$. Thus, the sequence $\frac{\overline{4n^2 - \iota n + 1}}{(\iota n+3)^2}$ is convergent.

3) $\frac{8^{n+1} - 5^n}{5 \cdot 8^n + 3^{n+1}} = \frac{8 - \left(\frac{5}{8}\right)^n}{5 + 3 \cdot \left(\frac{3}{8}\right)^n} \to \frac{8}{5}$ as $n \to \infty$, because $\left(\frac{5}{8}\right)^n \to 0$ as $n \to \infty$, $\left(\frac{3}{8}\right)^n \to 0$ as $n \to \infty$

Thus, the sequence $\frac{8^{n+1} - 5^n}{5 \cdot 8^n + 3^{n+1}}$ is convergent.