Answer on Question #49482 – Math – Complex Analysis

1) $\sum_{n=1}^{\infty}\overline{\left(\frac{1}{n^i}\right)}$ — test for convergence

Solution. Let us check the vanishing condition:

$\lim_{n\to\infty}\left|\overline{\left(\frac{1}{n^i}\right)}\right| = \lim_{n\to\infty}\left|\overline{\left(\frac{1}{e^{iLn(n)}}\right)}\right| = \lim_{n\to\infty}\left|\overline{e^{-iLn(n)}}\right| = \lim_{n\to\infty}\left|\overline{e^{-i(\ln n+i2\pi k)}}\right| = \lim_{n\to\infty}\left|e^{2\pi k}e^{i\ln n}\right| = e^{2\pi k}\neq 0$, where k is an integer constant. Vanishing condition is the necessary condition for summability. So, the series $\sum_{n=1}^{\infty}\overline{\left(\frac{1}{n^i}\right)}$ diverges.

Answer: the series diverges.

2) $\sum_{n=1}^{\infty}\frac{3i+n}{n^3+n+1}$ — test for convergence

Solution. Test it for absolute convergence:

$\sum_{n=1}^{\infty}\left|\frac{3i+n}{n^3+n+1}\right| = \sum_{n=1}^{\infty}\frac{\sqrt{n^2+9}}{n^3+n+1}$, use the comparison test $0\le\frac{\sqrt{n^2+9}}{n^3+n+1}\le\frac{\sqrt{10n^2}}{n^3}=\frac{\sqrt{10}}{n^2}$, but the series $\sum_{n=1}^{\infty}\frac{\sqrt{10}}{n^2}$ converges because the power of n in denominator is greater than 1. So, $\sum_{n=1}^{\infty}\frac{3i+n}{n^3+n+1}$ is absolutely convergent.

Answer: the series is absolutely convergent.

3) $\sum_{n=1}^{\infty}\left(\frac{1}{n^i}\right)^2$ — test for convergence

Solution. Let us check the vanishing condition:

$\lim_{n\to\infty}\left|\left(\frac{1}{n^i}\right)^2\right| = \lim_{n\to\infty}\left|\left(\frac{1}{e^{iLn(n)}}\right)^2\right| = \lim_{n\to\infty}\left|e^{-i2Ln(n)}\right| = \lim_{n\to\infty}\left|e^{-i2(\ln n+i2\pi k)}\right| = \lim_{n\to\infty}\left|e^{4\pi k}e^{-i2\ln n}\right| = e^{4\pi k}\neq 0$, where k is an integer constant. Vanishing condition is the necessary condition for summability. So, the series $\sum_{n=1}^{\infty}\left(\frac{1}{n^i}\right)^2$ diverges.

Answer: the series diverges.

4) $\sum_{n=1}^{\infty}e^{i\cosh n}$ — test for convergence

Solution. First of all, consider $e^{i\cosh n}$, n is an integer number, then $\cosh n=\frac{e^n+e^{-n}}{2}$ is real number, this means that $\left|e^{i\cosh n}\right|=1$

Let us check the vanishing condition:

$\lim_{n\to\infty}\left|e^{i\cosh n}\right|=1\neq 0$. Vanishing condition is the necessary condition for summability. So, the series $\sum_{n=1}^{\infty}e^{i\cosh n}$ diverges.

Answer: the series diverges.