Answer on Question #49463 – Math – Multivariable Calculus

$z = e^{xy}$; $x = 2u + v$; $y = \frac{u}{v}$;

Use a chain rule to find

a) ∂z/∂u
b) ∂z/∂v

Solution

$$\frac{\partial z}{\partial x} = ye^{xy}\ ; \quad \frac{\partial z}{\partial y} = xe^{xy}\ ;$$

$$\frac{\partial x}{\partial u} = 2\ ; \ \frac{\partial x}{\partial v} = 1;\ \frac{\partial y}{\partial u} = \frac{1}{v};\ \frac{\partial y}{\partial v} = -\frac{u}{v^2}\ ;$$

a)

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = 2ye^{xy} + \frac{1}{v}xe^{xy} = \left(2\frac{u}{v} + \frac{1}{v}(2u+v)\right)e^{(2u+v)\frac{u}{v}} = \left(4\frac{u}{v} + 1\right)e^{(2u+v)\frac{u}{v}}$$

b)

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = 1ye^{xy} - \frac{1}{v^2}xe^{xy} = \left(\frac{u}{v} - \frac{u}{v^2}(2u+v)\right)e^{(2u+v)\frac{u}{v}}$$