Answer on Question #49458 – Math – Multivariable Calculus

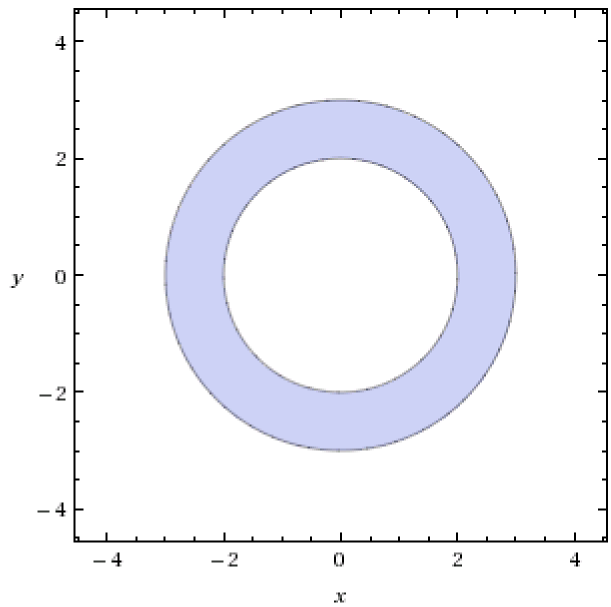
Given the triple integral

∭A(x^2+y^2)^3/2 dxdydz ,
A={(x,y,z):4≤x^2+y^2≤9, 1≤z≤2 }

Sketch the solid A and evaluate the integral

Solution

We have a solid: $A = \{(x, y, z): 4 \le x^2 + y^2 \le 9, 1 \le z \le 2\}$. In the xy-plane it looks like this



We can see that it is a solid that bounded by two cylinders: $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$ with height $h = z_2 - z_1 = 2 - 1 = 1$.

Let's evaluate the integral. Our integrand doesn't depends on z, so

$$I = \iiint_A (x^2 + y^2)^{3/2} dxdydz = \iint_S (x^2 + y^2)^{3/2} dxdy \int_1^2 dz = \iint_S (x^2 + y^2)^{3/2} dxdy$$

where $S = \{(x, y): 4 \le x^2 + y^2 \le 9\}$

We use polar coordinates:

$$\begin{cases} x = r\cos\varphi, \\ y = r\sin\varphi, \end{cases} \quad |J| = r$$

Then

$$(x^2 + y^2)^{3/2} = (r^2)^{\frac{3}{2}} = r^3$$

$$4 \le x^2 + y^2 \le 9 \;\rightarrow\; 4 \le r^2 \le 9 \;\rightarrow\; 2 \le r \le 3$$

Substitute it into I:

$$I = \iint_S (x^2 + y^2)^{\frac{3}{2}} dxdy = \int_0^{2\pi} \int_2^3 r^3 r dr d\varphi = \int_0^{2\pi} \int_2^3 r^4 dr d\varphi = 2\pi \frac{r^5}{5}\Big|_{r=2}^{r=3} = \frac{2\pi}{5}(3^5 - 2^5) =$$

$= \frac{2\pi}{5} \cdot 211 = \frac{422\pi}{5}$.

Answer: $I = \frac{422\pi}{5}$.