Answer on Question #49452 – Math – Multivariable Calculus

Find all the local maxima, local minima and saddle points of the function
f(x,y)=x^3+y^3+3x^2-2y^2-8

Solution.

$$f(x,y) = x^3 + y^3 + 3x^2 - 2y^2 - 8$$

$$f_x = 3x^2 + 6x, \; f_{xx} = 6(x+1),$$

$$f_y = 3y^2 - 4y, \quad f_{yy} = 2(3y-2), \qquad f_{xy} = 0.$$

$$D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = 12(x+1)(3y-2)$$

$$f_x = 0 \to \; 3x^2 + 6x = 0 \to \; x = 0 \; or \; x = -2$$

$$f_y = 0 \to \; 3y^2 - 4y = 0 \to \; y = 0 \; or \; y = \frac{4}{3}$$

So, we have 4 critical points: $(0,0)$, $\left(0,\frac{4}{3}\right)$, $(-2,0)$, $\left(-2,\frac{4}{3}\right)$. **Now, test them.**

$$D(0,0) = -24 < 0\,, therefore\; (0,0) - saddle\,point; f(0,0) = -8.$$

$$D\left(0,\frac{4}{3}\right) = 24 > 0 \; and \; f_{xx}\left(0,\frac{4}{3}\right) = 6 > 0, \qquad therefore$$

$$\left(0,\frac{4}{3}\right) - local\,minimum; \; f\left(0,\frac{4}{3}\right) = -\frac{248}{27}.$$

$$D(-2,0) = 24 > 0 \; and \; f_{xx}(-2,0) = -6 < 0, therefore$$

$$(-2,0) - local\,maximum; \; f(-2,0) = -4.$$

$$D\left(-2,\frac{4}{3}\right) = -24 < 0, therefore \; \left(-2,\frac{4}{3}\right) - saddle\,point;$$

$$f\left(-2,\frac{4}{3}\right) = -\frac{140}{27}.$$