Answer on Question #49451 – Math – Multivariable Calculus

Use the method of Lagrange multipliers to find the maximum and minimum values of the function f(x,y) = 3x + 4y on the circle x^2 + y^2 = 1

Solution.

$\boldsymbol{f(x,y) = 3x + 4y, \;\; g(x,y) = x^2 + y^2 - 1.}$

We must solve $\boldsymbol{\nabla f = \lambda \nabla g}$**. This equation is actually the two equations:**

$\boldsymbol{3 = 2\lambda x \quad and \;\; 4 = 2\lambda y}$ **along with the constraint equation** $\boldsymbol{x^2 + y^2 = 1}$

We have: $\boldsymbol{x = \frac{3}{2\lambda}, \;\; y = \frac{2}{\lambda},}$

$$\boldsymbol{substitute\ them\ into}$$

$$\boldsymbol{x^2 + y^2 = 1, multiply\ by\ \lambda^2\ and\ obtain\ \ \frac{9}{4} + 4 = \lambda^2.}$$

Thus, $\boldsymbol{\lambda = \pm\frac{5}{2}, \;\; x = \pm\frac{3}{5}, \;\; y = \pm\frac{4}{5}.}$

$\boldsymbol{f\left(\frac{3}{5},\frac{4}{5}\right) = 5\ and\ (x,y) = \left(\frac{3}{5},\frac{4}{5}\right) is\ a\ relative\ maximum.}$

$\boldsymbol{f\left(-\frac{3}{5},-\frac{4}{5}\right) = -5\ and\ \left(-\frac{3}{5},-\frac{4}{5}\right) is\ a\ relative\ minimum.}$