Answer on Question #49450 – Math – Linear Algebra

A floor manager is going to install two types of machine, small and large. The following table shows the number of operators and the space requirements for each machine:

	Small	Large	Maximum available
Number of operators	5	4	40
Space in m2	2	4	28

i) Taking x to represent the number of small machines and y to represent the number of large machines, write down two inequalities in x and y and illustrate these on a graph.

ii) If the profit on each small machine is €120 per day and the profit on each large machine is €200 per day, calculate the number of each type of machine that should be installed in order to have a maximum profit. What is the profit?

Solution

	Small	Large	Maximum available
Number of operators	5	4	40
Space in m2	2	4	28

i) Let x represent the number of small machines and y represent the number of large machines, the two inequalities in x and y will be the following:

$5x + 4y \leq 40$

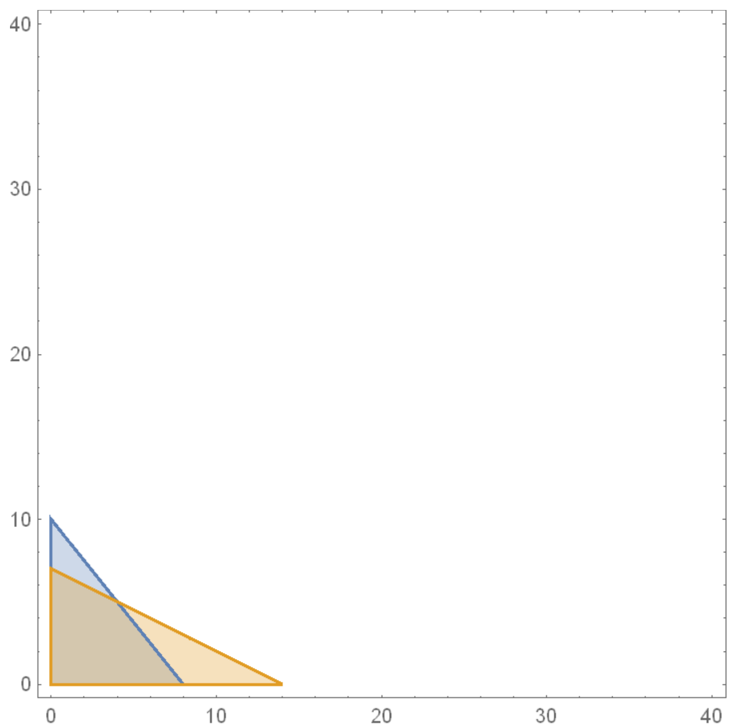
$2x + 4y \leq 28$

It is obvious that from practical point of view, we require that $x \geq 0, y \geq 0$.

Straight line $5x + 4y = 40$ is blue in the figure below, its x-intercept is (x, y) = (8,0), its y-intercept is (x, y) = (0,10).

Straight line $2x + 4y = 28$ is yellow in the figure below, its x-intercept is (x, y) = (14,0), its y-intercept is (x, y)=(0,7).

Two straight lines intersect at the point where $x = 4, y = 5$.



ii) If the profit on each small machine is €120 per day and the profit on each large machine is €200 per day, to calculate the number of each type of machine that should be installed in order to have a maximum profit

Method 1

Check values of function 120x + 200y under different cutting-edge points of region

$\{5x + 4y \leq 40,\ 2x + 4y \leq 28,\ x \geq 0,\ y \geq 0\}$:

(x,y)=(8,0), (x,y)=(0,7), (x,y)=(4,5)

If (x,y)=(8,0), then 120x + 200y=120*8=960 Euros

If (x,y)=(0,7), then 120x + 200y=200*7=1400 Euros

If (x,y)=(4,5), then 120x + 200y=120*4+200*5=1480 Euros

The maximum value of function 120x + 200y in the region $\{5x + 4y \leq 40,\ 2x + 4y \leq 28,\ x \geq 0,\ y \geq 0\}$ is attained when (x,y)=(4,5), it equals 1480 Euros.

Method 2

We use solver in excel.
The target cell is 120x + 200y = max
Restrictions are:
5x + 4y <=40
2x + 4y <=28
(x and y are empty cells)
The maximum profit is: 1480 Euros (when x=4, y=5).