Answer on Question # 49126 – Math – Calculus

Determine whether the series convergent or divergent and find its sum if exist

$$\sum_{n=1}^{\infty} \frac{5}{\sqrt{n}-3}.$$

Solution.

We have $\displaystyle\sum_{n=1}^{\infty} \frac{5}{\sqrt{n}-3}$

$$a_n = \frac{5}{\sqrt{n}-3}.$$

We use the integral test

Let $f(x) = \dfrac{5}{\sqrt{x}}$. Consider the integral $\displaystyle\int_1^{\infty} \frac{5}{\sqrt{x}} dx$.

$$\int_1^{\infty} \frac{5}{\sqrt{x}} dx = 5 \lim_{M\to\infty} \int_1^M x^{-1/2} dx = 5 \lim_{M\to\infty} \frac{x^{1/2}}{1/2}\Big|_1^M = 10 \lim_{M\to\infty} x^{1/2}\Big|_1^M =$$

$$= 10 \lim_{M\to\infty} \left(M^{1/2} - 1\right) = \infty.$$

Thus the integral $\displaystyle\int_1^{\infty} \frac{5}{\sqrt{x}} dx$ diverges.

Since $\dfrac{5}{\sqrt{x}-3} > \dfrac{5}{\sqrt{x}}$ then the integral $\displaystyle\int_1^{\infty} \frac{5}{\sqrt{x}-3} dx$ diverges too.

Hence by the Integral Test the series $\displaystyle\sum_{n=1}^{\infty} \frac{5}{\sqrt{n}-3}$ diverges.

Answer: The series $\displaystyle\sum_{n=1}^{\infty} \frac{5}{\sqrt{n}-3}$ diverges.