## Answer on Question #49123 – Math – Calculus

determine whether the series convergent or divergent and find its sum if exist $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{1}{n(n+1)}\right)$

**Solution.**

Denote $a_n=\frac{1}{2^n}, b_n=\frac{1}{n(n+1)}$.

We have $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}\frac{1}{2^n}$ is a geometric series. So,

$$\sum_{a_n}^{\infty} a_n = \frac{a_1}{1-q} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1;$$

Note that

$$b_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}.$$

Compute $\sum_{n=1}^{k} b_n$:

$$\sum_{n=1}^{k} b_n = b_1 + \dots + b_k = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{k} - \frac{1}{k+1} = 1 - \frac{1}{k+1}.$$

So:

$$\sum_{n=1}^{\infty} b_n = \lim_{k\to\infty}\sum_{n=1}^{k} b_n = \lim_{k\to\infty}\left(1-\frac{1}{k+1}\right) = 1;$$

So, the series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ is convergent. Because $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ are convergent series, compute the sum of the initial series $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{1}{n(n+1)}\right)$ :

$$\sum_{n=1}^{\infty}(a_n+b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n = 1+1 = 2.$$