

Answer on Question #49109, Math, Complex Analysis

$\int_C e^{z^*} dz$, where C -liner segment from $4i$ to 2 .

To count integral we will go by next way $4i \rightarrow 0 \rightarrow 2$

$$\int_2^{4i} = \int_{4i}^0 e^{z^*} dz + \int_0^2 e^{z^*} dz = - \int_0^{4i} e^{z^*} dz + \int_2^0 e^{z^*} dz$$

To count integral over Im axis we will use substitution $z^* = -it$ for the first integral and $z^* = t$ for the second and we will have the next condition:

$$i \int_{-4}^0 e^{-it} dt + \int_0^2 e^t dt = e^{-it} \Big|_{-4}^0 + e^t \Big|_0^2 = e^0 - e^{-4i} + e^2 - e^0 = e^2 - \cos 4 + i \sin 4$$

Explaining for first integral:

We make a substitution $z^* = -it \rightarrow dz^* = dz = -idt$

$$-i \int_0^{-4} e^{-it} dt = i \int_{-4}^0 e^{-it} dt = -i \cdot ie^{-it} \Big|_{-4}^0$$