

Answer on Question #49015 – Math – Statistics and Probability

I surveyed each house on the block to determine how much we are spending on Halloween treats. The distribution below represents the results in dollars.

\$Spent	f
50-59	1
40-49	2
30-39	5
20-29	12
10-19	7
0-9	7

Calculate the 90% confidence interval of the mean and present it in a sentence that shows your understanding of the meaning of the confidence interval.

Solution

The mean is

$$\mu = \frac{54.5 \cdot 1 + 44.5 \cdot 2 + 34.5 \cdot 5 + 24.5 \cdot 12 + 14.5 \cdot 7 + 4.5 \cdot 7}{1 + 2 + 5 + 12 + 7 + 7} = \frac{743}{34} = \$21.85.$$

Standard deviation is

$$\sigma = \sqrt{\frac{\sum fx^2 - n\mu^2}{n}}.$$

$$\sum fx^2 = 54.5^2 \cdot 1 + 44.5^2 \cdot 2 + 34.5^2 \cdot 5 + 24.5^2 \cdot 12 + 14.5^2 \cdot 7 + 4.5^2 \cdot 7 = 21698.5.$$

$$\sigma = \sqrt{\frac{21698.5 - 34 \cdot 21.85^2}{34}} = \$12.68.$$

The 90% confidence interval of the mean is

$$\begin{aligned} \left(\mu - z_{\frac{1-\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \mu + z_{\frac{1-\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right) &= \left(21.85 - z_{0.05} \frac{12.68}{\sqrt{34}}; 21.85 + z_{0.05} \frac{12.68}{\sqrt{34}} \right) \\ &= \left(21.85 - 1.645 \cdot \frac{12.68}{\sqrt{34}}; 21.85 + 1.645 \cdot \frac{12.68}{\sqrt{34}} \right) = (\$18.27; \$25.43). \end{aligned}$$

With 90% confidence level, the mean spent of the block lies in the interval (\$18.27; \$25.43).