Answer on Question #48982 – Math – Integral Calculus

1. Complete the perfect square and use tables of integration to integrate

$$\int \frac{1}{\sqrt{x^2 - 3x + 6}} dx$$

Solution:

$$\int \frac{1}{\sqrt{x^2 - 3x + 6}} dx = \int \frac{1}{\sqrt{x^2 - 3x + 2.25 + 3.75}} dx = \int \frac{1}{\sqrt{(x - 1.5)^2 + 3.75}} dx$$

Use tables of integration:

$$\int \frac{1}{\sqrt{x^2 + B}} dx = \ln\left|x + \sqrt{x^2 + B}\right| + C$$

$\int \frac{1}{\sqrt{(x-1.5)^2+3.75}} dx = \ln\left|(x - 1.5) + \sqrt{x^2 - 3x + 6}\right| + C$, C is an arbitrary real constant.

Answer:

$$\boldsymbol{\int \frac{1}{\sqrt{x^2 - 3x + 6}} dx = \ln\left|(x - 1.5) + \sqrt{x^2 - 3x + 6}\right| + C}$$

2. Use a substitution technique and then the table of integration to integrate

$$\int x^5\sqrt{x^4 - 4}dx$$

Solution:

$$\int x^5\sqrt{x^4 - 4}dx = \int x^7\sqrt{1 - \frac{4}{x^4}}dx$$

Let

$$1 - \frac{4}{x^4} = t^2, t = \frac{\sqrt{x^4 - 4}}{x^2}$$

$$x^4 = \frac{4}{1 - t^2}$$

$$4x^3dx = \frac{8t}{(1 - t^2)^2}dt$$

$$dx = \frac{2t}{x^3(1 - t^2)^2}dt$$

$$\int x^7\sqrt{1-\frac{4}{x^4}}dx = \int x^7 t\frac{2t}{x^3(1-t^2)^2}dt = 2\int x^4\frac{t^2}{(1-t^2)^2}dt = 8\int\frac{t^2}{(1-t^2)^3}dt$$

Use the partial fraction:

$$\frac{t^2}{(1-t^2)^3} = \frac{-t^2}{(t^2-1)^3} = \frac{A}{t+1} + \frac{B}{(t+1)^2} + \frac{C}{(t+1)^3} + \frac{D}{t-1} + \frac{E}{(t-1)^2} + \frac{F}{(t-1)^3}$$

$$A(t+1)^2(t-1)^3 + B(t+1)(t-1)^3 + C(t-1)^3 + D(t-1)^2(t+1)^3 +$$
$$+E(t-1)(t+1)^3 + F(t+1)^3 = -t^2$$

$$A = -\frac{1}{16}, B = -\frac{1}{16}, C = \frac{1}{8}, D = \frac{1}{16}, E = -\frac{1}{16}, F = -\frac{1}{8}$$

$$8\int\frac{t^2}{(1-t^2)^3}dt = \frac{1}{2}\int\left(-\frac{1}{t+1} - \frac{1}{(t+1)^2} + \frac{2}{(t+1)^3} + \frac{1}{t-1} - \frac{1}{(t-1)^2} - \frac{2}{(t-1)^3}\right)dt =$$

$$= \frac{1}{2}\left(\left(\ln\frac{t-1}{t+1}\right) + \frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{(t+1)^2} + \frac{1}{(t-1)^2}\right) + C =$$

$$= \frac{1}{2}\left(\ln\frac{t-1}{t+1}\right) + \frac{t}{(t^2-1)} - \frac{2t}{(t^2-1)^2} + C$$

$$t = \frac{\sqrt{x^4-4}}{x^2}$$

$$\int x^5\sqrt{x^4-4}dx = \frac{1}{2}\left(\ln\frac{\frac{\sqrt{x^4-4}}{x^2}-1}{\frac{\sqrt{x^4-4}}{x^2}+1}\right) + \frac{\frac{\sqrt{x^4-4}}{x^2}}{\left(\frac{x^4-4}{x^4}-1\right)} - \frac{2\frac{\sqrt{x^4-4}}{x^2}}{\left(\frac{x^4-4}{x^4}-1\right)^2} + C =$$

$= \frac{1}{2}\ln\frac{\sqrt{x^4-4}-x^2}{\sqrt{x^4-4}+x^2} - \frac{x^2\sqrt{x^4-4}}{8} + \frac{x^6\sqrt{x^4-4}}{4} + C,$ C is an arbitrary real constant.