

Answer on Question #48975 – Math – Calculus

A boat on the the ocean is 9 km from the nearest point on a straight shoreline; that point is 20 km from a restaurant on the shore. A woman plans to row the boat straight to a point on the shore and then walk along the shore to the restaurant. If she walks at 5 km/h and rows at 4 km/h. How far will the point on shore be from the restaurant if she plans to minimize her total travel time

Solution.

Let A be a boat, B – the nearest point on the shoreline, C – landing point, D – restorant.

Let $AB = w=9$, $BC = x$, $BD = y=20$, angle $BAC = z$

$$\text{Distance over water} = \frac{w}{\cos z} = \frac{9}{\cos z} .$$

$$\text{Distance over land} = 20 - w \tan z = 20 - 9 \tan z .$$

$$\text{Time } t = \frac{9}{4 \cos z} + \frac{(20 - 9 \tan z)}{5}$$

$$\frac{dt}{dz} = \frac{9 \sin z}{4 \cos^2 z} - \frac{9}{5 \cos^2 z} = 0 \rightarrow \sin z = \frac{4}{5} \rightarrow \cos z = \frac{3}{5} \rightarrow \tan z = \frac{4}{3}$$

$$\text{So, } x = 9 \tan z = 9 * \frac{4}{3} = 12$$

$$\text{Distance from restaurant} = y - x = 20 - 12 = 8 \text{ km.}$$