Answer on Question # 48974 – Math – Calculus

Determine whether the series convergent or divergent by using integral test

$$\sum_{n=2}^{\infty}\frac{2\ln(n-1)}{n-1}$$

Solution.

Consider the series $\sum_{n=2}^{\infty}\frac{2\ln(n-1)}{n-1}$ and integral $\int_2^{\infty}\frac{2\ln(x-1)}{x-1}dx$.

We use the integral test

$$\int_2^{\infty}\frac{2\ln(x-1)}{x-1}dx=\lim_{A\to\infty}\int_2^{A}\frac{2\ln(x-1)}{x-1}dx=\begin{bmatrix}\ln(x-1)=t\\ dt=\frac{dx}{x-1}\\ x=2\Rightarrow t=\ln 1=0\\ x=A\Rightarrow t=\ln(A-1)\end{bmatrix}=\lim_{A\to\infty}\int_0^{\ln(A-1)}2t\,dt=\lim_{A\to\infty}\left.\frac{2t^2}{2}\right|_0^{\ln(A-1)}=$$

$$\lim_{A\to\infty}\left.t^2\right|_0^{\ln(A-1)}=\lim_{A\to\infty}\left(\ln^2(A-1)-0\right)=\infty.$$

Hence by the Integral Test the series $\sum_{n=2}^{\infty}\frac{2\ln(n-1)}{n-1}$ diverges.

Answer: The series $\sum_{n=2}^{\infty}\frac{2\ln(n-1)}{n-1}$ diverges.