

Answer on Question #48852 – Math – Algebra

Show that the ratio of the sum of n terms of two G.P. having the same common ratio is equal to the ratio of their nth terms.

Solution.

G.P. 1: $a_1, a_1r, a_1r^2, \dots, a_1r^n$

G.P. 2: $b_1, b_1r, b_1r^2, \dots, b_1r^n$

The sum of n terms of G.P. 1 is

$$S_n^a = \frac{a_1(r^n - 1)}{r - 1}.$$

The sum of n terms of G.P. 2 is

$$S_n^b = \frac{b_1(r^n - 1)}{r - 1},$$

Ratio of the sum of n terms $\frac{S_n^a}{S_n^b} = \frac{a_1}{b_1}$

Ratio of their nth terms $\frac{a_1r^n}{b_1r^n} = \frac{a_1}{b_1}$.

The right hand sides ($\frac{a_1}{b_1}$) of these equalities are the same, therefore their left hand sides are equal too:

$$\frac{S_n^a}{S_n^b} = \frac{a_1r^n}{b_1r^n}.$$

Thus, the ratio of the sum of n terms equals the ratio of their nth terms.