Answer on Question #48849 – Math – Trigonometry

In a cyclic quadrilateral ABCD PROve that sin(a/2) =cos(b/2)

Note. ***Usually in a quadrilateral $ABCD$ the points A, B, C and D denote the successive vertices of the figure and we maintain this tradition in our discussion.***

Solution.

So the angles a and b are the neighboring angles of the figure and the problem statement for this reason is incorrect. Really, take into account that the sum of the opposite angles of the inscribed quadrilateral equals 180^0 and assume that BD is the diameter of a circle. Now let us choose the points A and C near the point B. Then each of the angles a and c is equal to 90^0 and by this reason $\sin(a/2) = \sin 45^0 = \frac{\sqrt{2}}{2}$. But the angle d is close to zero, and the value of the opposite angle b is about 180^0. Thus $\cos(b/2) \approx \cos 90^0 = 0 \neq \frac{\sqrt{2}}{2} = \sin(a/2)$ and the desired statement is not satisfied.

Note. The statement of the problem should look like this: sin(a/2)=cos(c/2).

Indeed, since a and c are the opposite angles of the inscribed quadrilateral then $a + c = 180^0$ then $\frac{a}{2} + \frac{c}{2} = 90^0$ then $\frac{a}{2} = 90^0 - \frac{c}{2}$ then $\sin\frac{a}{2} = \sin(90^0 - \frac{c}{2})$ or $\sin\frac{a}{2} = \cos\frac{c}{2}$ .Q.E.D.