Answer on Question #48818 – Math – Complex Analysis

Solve the equation

1) sin z+ cos z=0

2) exp(z^2) = exp 3z ; Im z in (-pi,pi]

3) ln(z+1)=ln i

Solution.

1) $\sin z = \dfrac{e^{iz} - e^{-iz}}{2i}, \quad \cos z = \dfrac{e^{iz} + e^{-iz}}{2}.$

Let denote $e^{iz} = t$. Then $\sin z = \dfrac{1}{2i}\left(t - \dfrac{1}{t}\right)$, $\cos z = \dfrac{1}{2}\left(t + \dfrac{1}{t}\right)$.

$$\frac{1}{2i}\left(t - \frac{1}{t}\right) + \frac{1}{2}\left(t + \frac{1}{t}\right) = 0, \quad t^2 - 1 + i(t^2 + 1) = 0, \quad t^2 = \frac{1-i}{1+i} = \frac{(1-i)^2}{1^2 + 1^2} = \frac{1 - 2i - 1}{2} = -i.$$

$$e^{2iz} = -i = 1 \cdot e^{-i\frac{\pi}{2}}, \quad 2iz = -i\frac{\pi}{2} + 2\pi k i, \quad z = -\frac{\pi}{4} + \pi k, \quad k \in Z.$$

2) $e^{z^2} = e^{3z}, \quad z^2 = 3z + 2\pi k i, \quad z^2 - 3z - 2\pi k i = 0, \quad k \in Z.$

$$D = 3^2 + 4 \cdot 2\pi k i = 9 + 8\pi k i. \quad z = \frac{3 \pm \sqrt{D}}{2} = \frac{3 \pm \sqrt{81 + 64\pi^2 k^2}\, e^{i\varphi}}{2},$$

where $\varphi = \left\{ \dfrac{1}{2} arctg \dfrac{8\pi k}{9}; \dfrac{1}{2} arctg \dfrac{8\pi k}{9} + \pi \right\}$.

As $\operatorname{Im} z \in (-\pi; \pi]$, then $\dfrac{\sqrt{81 + 64\pi^2 k^2}}{2} \le \pi$. None of $k \in Z$ obeys this inequality, because $\dfrac{\sqrt{81 + 64\pi^2 \cdot 0^2}}{2} = \dfrac{9}{2} > \pi$.

So, there is no solution.

3) $\ln(z+1) - \ln i = 0, \quad \ln\dfrac{z+1}{i} = 0, \quad \dfrac{z+1}{i} = 1, \quad z = i - 1.$

Answer: 1) $z = -\dfrac{\pi}{4} + \pi k, \quad k \in Z$; 2) no solution; 3) $z = i - 1$.