Answer on Question #48726 – Math – Statistics and Probability

Let $f_X(x) = \frac{1}{3} x[U(x) - U(x-2)] + \frac{1}{3} \delta(x-1)$. Find

The expected value E[X]. B. the variance σx^2.

Solution

$$\mathrm{E[X]} = \int_{-\infty}^{\infty} x f_X(x) dx = \int_{-\infty}^{\infty} x \cdot \left(\frac{1}{3} x[U(x) - U(x-2)] + \frac{1}{3} \delta(x-1) \right) dx$$

$$= \frac{1}{3} \int_{-\infty}^{\infty} x^2 \cdot [U(x) - U(x-2)] dx + \frac{1}{3} \int_{-\infty}^{\infty} x \cdot \delta(x-1) dx.$$

$$\int_{-\infty}^{\infty} x \cdot \delta(x-1) dx = 1.$$

$$\int_{-\infty}^{\infty} x^2 \cdot [U(x) - U(x-2)] dx = \int_{0}^{2} x^2 dx = \left(\frac{x^3}{3} \right)_0^2 = \frac{8}{3}.$$

So, the expected value is

$$\mathrm{E[X]} = \frac{1}{3} \cdot \frac{8}{3} + \frac{1}{3} \cdot 1 = \frac{11}{9}.$$

The variance is

$$\sigma_x^2 = \mathrm{E[X^2]} - (\mathrm{E[X]})^2, \text{ where}$$

$$\mathrm{E[X^2]} = \int_{-\infty}^{\infty} x^2 f_X(x) dx = \int_{-\infty}^{\infty} x^2 \cdot \left(\frac{1}{3} x[U(x) - U(x-2)] + \frac{1}{3} \delta(x-1) \right) dx$$

$$= \frac{1}{3} \int_{-\infty}^{\infty} x^3 \cdot [U(x) - U(x-2)] dx + \frac{1}{3} \int_{-\infty}^{\infty} x^2 \cdot \delta(x-1) dx.$$

$$\int_{-\infty}^{\infty} x^2 \cdot \delta(x-1) dx = 1^2 = 1.$$

$$\int_{-\infty}^{\infty} x^3 \cdot [U(x) - U(x-2)] dx = \int_{0}^{2} x^3 dx = \left(\frac{x^4}{4} \right)_0^2 = \frac{2^4}{4} = 4.$$

So, $\mathrm{E[X^2]} = \frac{1}{3} \cdot 4 + \frac{1}{3} \cdot 1 = \frac{5}{3}$.

Thus,

$$\sigma_x^2 = \frac{5}{3} - \left(\frac{11}{9} \right)^2 = \frac{5 \cdot 27 - 121}{81} = \frac{14}{81}$$