Answer on Question #48694 – Math – Algebra

Two rooms in a very large building are connected by an archway. The height, in feet, of the archway x feet from one side of the opening is given by the function.

$$f(x) = -3.3x^2 + 27x$$

How wide is the archway? How tall is the archway at its highest point?

The archway is 8.182 feet wide and 55.227 feet tall.

Solution:

In given problem we need to consider the quadratic equation in order to find the value of width and height of the archway.

We know that the Maximum height is at the vertex. For a parabola in form $y = f(x) = ax^2 + bx + c$, the vertex is located at $(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right))$. In our task we have x is the independent variable, so we can note that a = -3.3 and b = 27.

We can find the value of vertex by substituting the given values according to the condition of the task.

$$-\frac{b}{2a} = -\frac{27}{(2 \cdot (-3.3))} = \frac{27}{6.6} = 4.091 \text{ feet}$$

Now we search for the function in order to find the highest point.

$$f\left(-\frac{b}{2a}\right) = f(4.091) = -3.3 \cdot (4.091)^2 + 27 \cdot 4.091 = 55.227 \text{ feet}$$

Therefore we conclude that maximum height is 55.227 feet.

The base is the horizontal axis, so the parabola intersects the base at the roots of

$$-3.3x^2 + 27x = 0$$

We can apply the factoring in this case to find the roots.

$$-x(3.3x - 27) = 0$$

The first root is equal to $x = 0$. Now we can find the second root.

$$3.3x - 27 = 0$$

We add 27 to both sides of the equation.

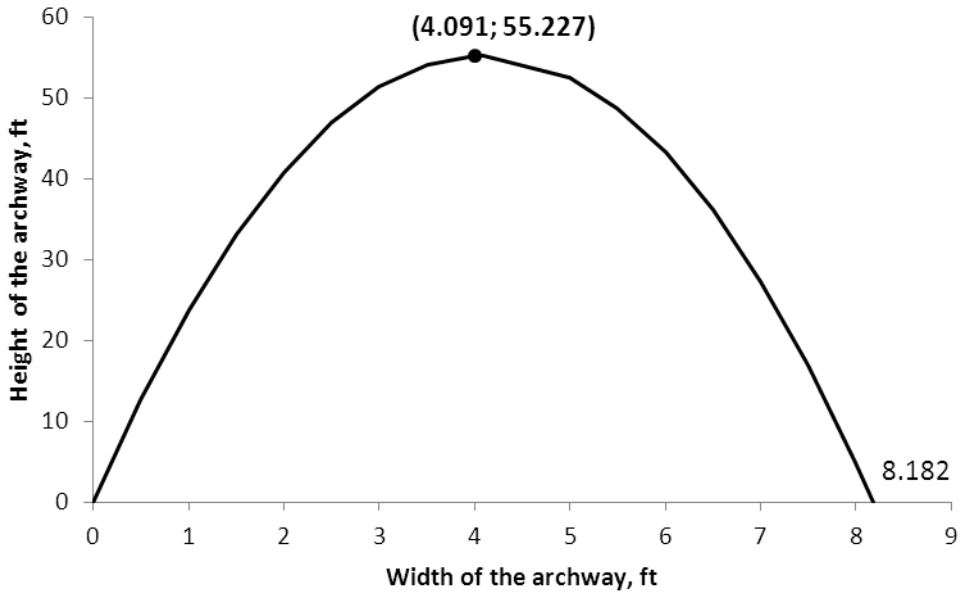
$$3.3x = 27$$

Now we divide both sides of the equation by 3.3.

$$x = 8.182$$

Hence $x = 0$ or $-3.3x^2 + 27x = 0$ => x=8.182, so the coordinates of the intersection of the parabola with the base on the x-h plane are (0,0) and (8.182,0).

We also can represent the graph of the construction on the chart.



The width of the archway is equal to 8.182 feet and the height is equal to 55.227 feet.