Answer on Question #48643 – Math – Calculus

1. Find second order derivative of $y=\log(\log x)$.

Solution.

$$y'=(\log(\log x))'=\frac{1}{\log x}(\log x)'=\frac{1}{\log x}\frac{1}{x}=\frac{1}{x\log x}$$

$$y''=\left(\frac{1}{x\log x}\right)'=\left((x\log x)^{-1}\right)'=-(x\log x)^{-2}(x\log x)'=-(x\log x)^{-2}\left(x'\log x+x(\log x)'\right)=$$
$$=-(x\log x)^{-2}\left(\log x+x\frac{1}{x}\right)=-(x\log x)^{-2}(\log x+1)=-\frac{1+\log x}{(x\log x)^2}$$

Answer: $-\frac{1+\log x}{(x\log x)^2}$

2. Find second order derivative of $y=\sin(\log x)$.

Solution.

$$y'=(\sin(\log x))'=\cos(\log x)(\log x)'=\cos(\log x)\frac{1}{x}=\frac{\cos(\log x)}{x}$$

$$y''=\left(\frac{\cos(\log x)}{x}\right)'=\frac{(\cos(\log x))'x-\cos(\log x)x'}{x^2}=\frac{\left(-\sin(\log x)\frac{1}{x}\right)x-\cos(\log x)}{x^2}=$$
$$=\frac{-\sin(\log x)-\cos(\log x)}{x^2}=-\frac{\sin(\log x)+\cos(\log x)}{x^2}.$$

Answer: $-\frac{\sin(\log x)+\cos(\log x)}{x^2}.$

3. Find second order derivative of $y=\frac{\log x}{x}$.

Solution.

$$y'=\left(\frac{\log x}{x}\right)'=\frac{(\log x)'x-\log x x'}{x^2}=\frac{\frac{1}{x}x-\log x}{x^2}=\frac{1-\log x}{x^2}.$$

$$y''=\left(\frac{1-\log x}{x^2}\right)'=\frac{(1-\log x)'x^2-(1-\log x)(x^2)'}{x^4}=\frac{\left(-\frac{1}{x}\right)x^2-2x(1-\log x)}{x^4}=$$
$$=\frac{-x-2x+2x\log x}{x^4}=\frac{-3x+2x\log x}{x^4}=\frac{-3+2\log x}{x^3}.$$

Answer: $\frac{-3+2\log x}{x^3}$.

4. Find second order derivative of $y=x^2\log|\cos x|$.

Solution.

$$y'=\left(x^2\log|\cos x|\right)'=\left(x^2\right)'\log|\cos x|+x^2\left(\log|\cos x|\right)'=2x\log|\cos x|+\frac{x^2}{|\cos x|}\left(|\cos x|\right)'=$$
$$=\begin{cases}2x\log|\cos x|+\frac{x^2}{\cos x}(\cos x)', & \text{if} \quad \cos x\ge 0\\ 2x\log|\cos x|+\frac{x^2}{-\cos x}(-\cos x)', & \text{if} \quad \cos x<0\end{cases}=$$
$$=\begin{cases}2x\log|\cos x|+\frac{-\sin x\cdot x^2}{\cos x}, & \text{if} \quad \cos x\ge 0\\ 2x\log|\cos x|+\frac{\sin x\cdot x^2}{-\cos x}, & \text{if} \quad \cos x<0\end{cases}=2x\log|\cos x|-x^2\tan x.$$

$$y''=\left(2x\log|\cos x|-x^2\tan x\right)'=2x'\log|\cos x|+2x\left(\log|\cos x|\right)'-\left(x^2\right)'\tan x-x^2(\tan x)'=$$
$$=2\log|\cos x|+2x(-\tan x)-2x\tan x-\frac{x^2}{\cos^2 x}=2\log|\cos x|-\frac{x^2}{\cos^2 x}-4x\tan x.$$

Answer: $2\log|\cos x|-\frac{x^2}{\cos^2 x}-4x\tan x$.

5. Find second order derivative of $y=\frac{2x+1}{2x+3}$.

Solution.

$$y'=\left(\frac{2x+1}{2x+3}\right)'=\frac{(2x+1)'(2x+3)-(2x+3)'(2x+1)}{(2x+3)^2}=\frac{2(2x+3)-2(2x+1)}{(2x+3)^2}=$$
$$=\frac{4x+6-4x-2}{(2x+3)^2}=\frac{4}{(2x+3)^2}$$

$$y'' = \left(\frac{4}{(2x+3)^2} \right)' = 4\left((2x+3)^{-2} \right)' = 4 \cdot (-2)(2x+3)^{-3} (2x+3)' = 4 \cdot (-2)(2x+3)^{-3} \cdot 2 = -\frac{16}{(2x+3)^3}.$$

Answer: $-\frac{16}{(2x+3)^3}.$