1. Find second order derivative of $y = \sin^{-1} x$.

Solution.

$$y' = \left(\sin^{-1} x\right)' = -\sin^{-1-1} x \left(\sin x\right)' = -\sin^{-2} x \cos x = -\frac{\cos x}{\sin^2 x}.$$

$$y'' = \left(-\frac{\cos x}{\sin^2 x}\right)' = -\frac{\left(\cos x\right)' \sin^2 x - \cos x \left(\sin^2 x\right)'}{\sin^4 x} = -\frac{-\sin x \sin^2 x - \cos x 2 \sin x \left(\sin x\right)'}{\sin^4 x} =$$

$$= -\frac{-\sin^3 x - 2\cos x \sin x \cos x}{\sin^4 x} = \frac{\sin^3 x + 2\cos^2 x \sin x}{\sin^4 x} = \frac{\sin^2 x + 2\cos^2 x}{\sin^3 x} =$$

$$= \frac{1 + \cos^2 x}{\sin^3 x}.$$

Answer: $\dfrac{1 + \cos^2 x}{\sin^3 x}.$

2. Find second order derivative of $y = x \cos x$.

Solution.

$$y' = \left(x \cos x\right)' = x' \cos x + x \left(\cos x\right)' = \cos x - x \sin x.$$

$$y'' = (cosx - xsinx)' = -sinx - sinx - xcosx = -2sinx - xcosx$$

Other method uses Leibnitz rule:

$$y'' = x'' cosx + 2x' (cosx)' + x(cosx)'' = 0 - 2sinx - xcosx =$$
$$= -2sinx - xcosx$$

Answer: $-2sinx - xcosx$

3. Find second order derivative of $y = e^{6x} \cos 3x$.

Solution.

$$y' = \left(e^{6x} \cos 3x\right)' = \left(e^{6x}\right)' \cos 3x + e^{6x} \left(\cos 3x\right)' = e^{6x} \left(6x\right)' \cos 3x + e^{6x} \left(-\sin 3x\right)\left(3x\right)' =$$
$$= 6e^{6x} \cos 3x - 3e^{6x} \sin 3x.$$

$$y'' = \left(6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right)' = 6\left(6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right) - 3\left(e^{6x}\sin 3x\right)' =$$

$$= 6\left(6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right) - 3\left(\left(e^{6x}\right)'\sin 3x + e^{6x}\left(\sin 3x\right)'\right) =$$

$$= 6\left(6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right) - 3\left(e^{6x}\left(6x\right)'\sin 3x + e^{6x}\cos 3x\left(3x\right)'\right) =$$

$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 3\left(6e^{6x}\sin 3x + 3e^{6x}\cos 3x\right) = 27e^{6x}\cos 3x - 36e^{6x}\sin 3x.$$

Other method uses Leibnitz rule:

$$y'' = (e^{6x}\cos 3x)'' = (e^{6x})''\cos(3x) + 2(e^{6x})'\,(\cos 3x)' + e^{6x}(\cos(3x))''$$
$$= 36e^{6x}\cos(3x) - 2\cdot 6\cdot e^{6x}\cdot 3\sin(3x) - 9e^{6x}\cos(3x) =$$
$$= 27e^{6x}\cos(3x) - 36e^{6x}\sin(3x)$$

Answer: $27e^{6x}\cos 3x - 36e^{6x}\sin 3x.$