Answer on Question #48624 – Math – Statistics and Probability

Lotto. A previous Arizona State lottery, called lotto, was played as follows: The player selects six numbers from the numbers 1-42 and buys a ticket for $1. There are six winning numbers, which are selected at random from the numbers 1-42. To win a prize, a lotto ticket must contain three or more of the winning numbers. A ticket with exactly three winning numbers is paid $2. The prize for a ticket with exactly four, five or six winning numbers depends on sales and on how many other tickets were sold that have exactly four, five or six winning numbers, respectively. If you buy one Lotto ticket, determine the probability that

a) you win the jackpot; that is , your six numbers are the same as the six winning numbers.

b) your ticket contains exactly four winning numbers.

c) you don't win a prize.

Solution

We have binomial distribution with $p = \frac{6}{42} = \frac{1}{7}$ and n=6.

a)

$$P(X = 6) = \frac{6!}{6!\,(6-6)!}\left(\frac{1}{7}\right)^6\left(\frac{6}{7}\right)^{6-6} = \left(\frac{1}{7}\right)^6 \approx 8.5 \cdot 10^{-6}.$$

b)

$$P(X = 4) = \frac{6!}{4!\,(6-4)!}\left(\frac{1}{7}\right)^4\left(\frac{6}{7}\right)^{6-4} = 15 \cdot 6^2\left(\frac{1}{7}\right)^6 \approx 0.0046.$$

c)

$$\begin{aligned} P(X < 3) &= P(X = 0) + P(X = 1) + P(X = 2) \\ &= \frac{6!}{0!\,(6-0)!}\left(\frac{1}{7}\right)^0\left(\frac{6}{7}\right)^{6-0} + \frac{6!}{1!\,(6-1)!}\left(\frac{1}{7}\right)^1\left(\frac{6}{7}\right)^{6-1} + \frac{6!}{2!\,(6-2)!}\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^{6-2} \\ &= \left(\frac{6}{7}\right)^6 + \left(\frac{6}{7}\right)^6 + 15 \cdot 6^4\left(\frac{1}{7}\right)^6 \approx 0.9584 \end{aligned}$$