Find dy/dx when sin(x+y)=2/3

Solution.

$$\sin(x + y) = \frac{2}{3}$$

Let's take a derivative of both parts d/dx :

$$\frac{d}{dx}\left(\sin(x + y)\right) = \frac{d}{dx}\left(\frac{2}{3}\right) = 0$$

$$\frac{d}{dx}\left(\sin(x + y)\right) = \cos(x + y) \cdot \frac{d}{dx}(x + y) = \cos(x + y) \cdot \left(1 + \frac{dy}{dx}\right)$$

So:

$$\cos(x + y) \cdot \left(1 + \frac{dy}{dx}\right) = 0$$

But:

$$\sin(x + y) = \frac{2}{3} \rightarrow \cos(x + y) = \sqrt{1 - \sin^2(x + y)} = \sqrt{1 - \left(\frac{2}{3}\right)^2} = \pm\frac{5}{9} \neq 0$$

So:

$$\left(1 + \frac{dy}{dx}\right) = 0$$

Or:

$$\frac{dy}{dx} = -1$$

Answer:
$$\frac{dy}{dx} = -1$$