Find dy/dx when $y \sec x + \tan x + x^2 y = 0$.

Solution:

Differentiating both sides:

$$\sec x \frac{dy}{dx} + y \frac{\sin x}{\cos^2 x} + \frac{1}{\cos^2 x} + 2xy + x^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(\sec x + x^2) = -y \frac{\sin x}{\cos^2 x} - \frac{1}{\cos^2 x} - 2xy$$

$$\frac{dy}{dx} = -\frac{y \dfrac{\sin x}{\cos^2 x} + \dfrac{1}{\cos^2 x} + 2xy}{\sec x + x^2} = -\frac{y \sin x + 2xy \cos^2 x + 1}{\cos x + x^2 \cos^2 x}$$

Answer:

$$\frac{dy}{dx} = -\frac{y \sin x + 2xy \cos^2 x + 1}{\cos x + x^2 \cos^2 x}$$